PAR-003-1015003

Seat No. ______

B. Sc. (Sem. V) (CBCS) (W.I.F. 2016) Examination

October / November - 2018

Mathematics : Math - 07 - A

(Boolean Algebra & Complex Analysis-I)

(Old Course)

Faculty Code : 003

Subject Code : 1015003

Time : **3** Hours] [Total Marks : **70**

Instructions : (1) All the questions are compulsory.
(2) Numbers written to the right indicate full marks of the question.

1 (a) Attempt all the questions : **4**

(1) Define antisymmetric relation.

(2) Give an example of an relation on {1, 2, 3} which is reflexive but not symmetric.

(3) Is (S_{30}, D) a chain ? Justify your answer.

(4) Write maximal elements for the POSET $(\{1, 2, 3, 4, 5, 6\}, /)$.

(b) Attempt any one : **2**

(1) State and prove Isotonicity property.

(2) State and prove Absorption law for a lattice.

(c) Attempt any **one** : **3**

(1) State and prove Modulus inequality.

(2) State and prove Distributive inequalities.

(d) Attempt any **one** : **5**

(1) Define direct product of lattices. Prove that direct product of two lattices is also a lattice

(2) In usual notation prove that in a complemented distributive lattice the following are equivalent.

(i) $a \le b$ (ii) $a \wedge b' = 0$

(iii) $a' \vee b = 1$ (iv) $b' \le a'$

2 (a) Attempt all the questions : **4**

(1) Define Boolean algebra.

(2) How many squares are there in a K-map of the expression contains 4-variables.

(3) Define atoms of a Boolean algebra.

(4) State unique representation theorem.

(b) Attempt any **one** : **2**

(1) In usual notation prove that in a Boolean algebra $(B, *, \oplus, ', 0, 1)$ for every a, b, c in $B (a*b) \oplus (b*c) \oplus (c*a) = (a \oplus b) * (b \oplus c) * (c \oplus a)$

(2) Obtain the minimal sum of product of the Boolean expression $\alpha(x, y, z) = xyz + xyz' + x'yz + x'y'z$ by Karnaugh map.

(c) Attempt any **one** : **3**

(1) In usual notation prove that a non zero element of a Boolean algebra $(B, *, \oplus, ', 0, 1)$ is an atom if and only if $\forall x \in B$ either $a * x = 0$ or $a * x = a$.

(2) Express as the product of its maxterm : $a(x_1, x_2, x_3) = (x_2 * x_3)$.

(d) Attempt any **one** : **5**

(1) State and prove Stone's representation theorem.

(2) Define minterms and in usual in notation prove that sum of all minterms of n-variable $x_1, x_2, ... x_n$ is 1.

3 (a) Attempt all the questions : **4**

(1) Define analytic function.

(2) Evaluate : $\lim_{z \to \infty} \frac{2z+3}{z+i}$

(3) Define Laplace equation.

(4) Define entire function.

(b) Attempt any **one** : **2**

(1) Prove that $exp(z)$ is analytic.

(2) Show that $f(z) = \overline{z}$ is not analytic function.

(c) Attempt any **one** : **3**

(1) In usual notation prove that if the complex function and its conjugate are analytic then it is a constant function.

(2) If u and v are conjugate harmonic function then prove that the family of curve obtained by $u = c_1$ and $v = c_2$ are orthogonal.

(d) Attempt any **one** : **5**

(1) Obtain necessary conditions for a complex function $f(z)$ to be analytic.

(2) Find an analytic function $f(z) = u + iv$ such that $u - v = x + y$.

4 (a) Attempt **all** the questions : **4**

(1) Write Laplace equation in polar form.

(2) Define Jorden arc.

(3) State Cauchy integral formula.

(4) Define Contour.

(b) Attempt any **one** : **2**

(1) Prove that $\int_0^{1+i} \overline{z}dz = 1$.

(2) Define closed curve and closed contour.

(c) Attempt any **one** : **3**

(1) State and prove Cauchy's fundamental theorem.

(2) Find $\int_C \frac{dz}{\left(z^2+4\right)^2}, C : |z - i| = 2$

(d) Attempt any **one** : **5**

(1) Obtain C-R conditions for analytic function $f(z)$ in polar form.

(2) Prove that $u = r^2 \sin 2\theta$ is a harmonic function and find its conjugate.

5 (a) Attempt **all** the questions : **4**

(1) Define : Limit of complex function.

(2) Evaluate : $\int_C \frac{(x^3+3)}{x} dz$ where C is an ellipse with center at origin.

(3) Which curve is represented by the equation $|z - z_0| = r_0$.

(4) Define : Complex function.

(b) Attempt any one : **2**

(1) State and prove Cauchy inequality.

(2) Evaluate $\int_C \frac{z}{(z-2)\left(z+\frac{1}{2}\right)} dz; c : |z| = 1$.

(c) Attempt any **one** : **3**

(1) State and prove Liouville's theorem.

(2) State and prove fundamental theorem of algebra.

(d) Attempt any **one** : **5**

(1) State and prove Morera's theorem.

(2) Evaluate : $\int_C \frac{z}{(2z-1)(z+1)} dz, C : |z| = 2$.
